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Square root of Binomial Surd.

$7 + 4\sqrt{3}$

Q no. find the square root of $5 + 2\sqrt{6}$

Sol: $\sqrt{a + \sqrt{b}} = \sqrt{x} + \sqrt{y}$ $\qquad 5 + 2\sqrt{6} = 5 + \sqrt{24}$

Method-1

$$\sqrt{5 + 2\sqrt{6}} = \sqrt{5 + \sqrt{24}}$$

$$\Rightarrow \sqrt{5 + \sqrt{24}} = \sqrt{2} + \sqrt{3}$$

or

$$= \sqrt{3} + \sqrt{2}$$

$$\sqrt{5 + \sqrt{24}} = \sqrt{x} + \sqrt{y}$$

squaring both side

$$\left(\sqrt{5 + \sqrt{24}}\right)^2 = \left(\sqrt{x} + \sqrt{y}\right)^2$$

$$\Rightarrow 5 + \sqrt{24} = (\sqrt{x})^2 + (\sqrt{y})^2 + 2(\sqrt{x})(\sqrt{y})$$

$$5 + \sqrt{24} = x + y + 2\sqrt{xy}$$

$$x + y = 5 \quad \ldots (1)$$

$$2\sqrt{xy} = \sqrt{24}$$

$$2\sqrt{xy} = 2\sqrt{6}$$

$$xy = 6 \quad \ldots (2)$$

$$\Rightarrow y = \frac{6}{x}$$

$\Rightarrow \boxed{y = \frac{6}{x}}$

$$x + \frac{6}{x} = 5$$

$$\Rightarrow \frac{x^2 + 6}{x} = 5 \quad \Rightarrow \quad x^2 + 6 = 5x$$

$$\Rightarrow x^2 - 5x + 6 = 0$$

$$\Rightarrow$$

$$x^2 - 2x - 3x + 6 = 0$$

$$x(x-2) - 3(x-2) = 0$$

$$(x-2)(x-3) = 0$$

either

$$x - 2 = 0 \quad \text{or} \quad x - 3 = 0$$

$$\boxed{x = 2} \quad \text{or} \quad \boxed{x = 3}$$

As $y = \frac{6}{x}$

If $x = 2$ $\quad y = \frac{6}{2} = 3$

If $x = 3$ $\quad y = \frac{6}{3} = 2$

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find $\sqrt{a+\sqrt{b}}$

let $\sqrt{a+\sqrt{b}} = \sqrt{x}+\sqrt{y}$

squaring both sides.

$a+\sqrt{b} = x+y+2\sqrt{xy}$

$x+y=a$ and $\sqrt{b} = 2\sqrt{xy}$ .... (1)

$\sqrt{4xy} = \sqrt{b}$

$\Rightarrow 4xy = b$

$\Rightarrow y = \frac{b}{4x}$ ... (2)

As $x+y=a$

$x+\frac{b}{4x} = a$

$\Rightarrow \frac{4x^2+b}{4x} = a$

$\Rightarrow 4x^2+b = 4ax$

$\Rightarrow 4x^2-4ax+b=0$ → Quadratic equation in terms 'x'

note:

$Ax^2+Bx+C=0$

$x = \frac{-B \pm \sqrt{B^2-4AC}}{2A}$

$A=4 \quad B=-4a \quad C=b$

$\Rightarrow x = \frac{-B \pm \sqrt{B^2-4AC}}{2A}$

$x = \frac{4a \pm \sqrt{16a^2-4\times4\times b}}{2\times4}$

$x = \frac{4a \pm \sqrt{16a^2-16b}}{8} = \frac{4a \pm 4\sqrt{a^2-b}}{8} = \frac{a \pm \sqrt{a^2-b}}{2}$

$$x = \frac{4a \pm \sqrt{16a^2 - 16b}}{8} = \frac{4a \pm 4\sqrt{a^2 - b}}{8} = \frac{a \pm \sqrt{a^2 - b}}{2}$$

$$x = \frac{a + \sqrt{a^2 - b}}{2} \quad \text{or} \quad x = \frac{a - \sqrt{a^2 - b}}{2}$$

As $x + y = a$

$y = a - x$

$y = a - \left(\frac{a + \sqrt{a^2 - b}}{2}\right)$

$y = \frac{2a - a - \sqrt{a^2 - b}}{2}$

$y = \frac{a - \sqrt{a^2 - b}}{2}$

As $x + y = a$

$y = a - x$

$= a - \left(\frac{a - \sqrt{a^2 - b}}{2}\right)$

$= \frac{2a - a + \sqrt{a^2 - b}}{2}$

$y = \frac{a + \sqrt{a^2 - b}}{2}$

$$\sqrt{a + \sqrt{b}} = \sqrt{x} + \sqrt{y}$$

$$\sqrt{a + \sqrt{b}} = \sqrt{\frac{a + \sqrt{a^2 - b}}{2}} + \sqrt{\frac{a - \sqrt{a^2 - b}}{2}}$$

example $5 + 2\sqrt{6} = 5 + \sqrt{24}$ ($a = 5$, $b = 24$)

$$\sqrt{5 + 2\sqrt{6}} = \sqrt{\frac{5 + \sqrt{25 - 24}}{2}} + \sqrt{\frac{5 - \sqrt{25 - 24}}{2}} = \sqrt{\frac{5+1}{2}} + \sqrt{\frac{5-1}{2}} = \sqrt{3} + \sqrt{2}$$

example $5 + 2\sqrt{6} = 3 + 2 + 2\sqrt{3 \times 2} = (\sqrt{3})^2 + (\sqrt{2})^2 + 2(\sqrt{3})(\sqrt{2}) = (\sqrt{3} + \sqrt{2})^2$

($3 + 2$, 3×2; a^2, b^2, $2ab$)

$$\sqrt{5+2\sqrt{6}} = \sqrt{(\sqrt{3}+\sqrt{2})^2} = \sqrt{3}+\sqrt{2}$$

eg. $7+4\sqrt{3} = 7+2\times 2\sqrt{3} = 7+2\sqrt{12} = 4+3+2\sqrt{4\times 3} = \sqrt{4}+\sqrt{3}$

$= 2+\sqrt{3}$

($7 = 4+3$, $12 = 4\times 3$)

eg. $2+\sqrt{3} = 2+\frac{2}{2}\sqrt{3} = 2+2\sqrt{\frac{3}{4}} = \frac{3}{2}+\frac{1}{2}+2\sqrt{\frac{3}{2}\cdot\frac{1}{2}}$

($2 = \frac{3}{2}+\frac{1}{2}$, $\frac{3}{4} = \frac{1}{2}\times\frac{3}{2}$)

$= \left(\sqrt{\frac{3}{2}}\right)^2 + \left(\sqrt{\frac{1}{2}}\right)^2 + 2\sqrt{\frac{3}{2}}\cdot\sqrt{\frac{1}{2}}$

$= \left(\sqrt{\frac{3}{2}}+\sqrt{\frac{1}{2}}\right)^2$

$$\sqrt{2+\sqrt{3}} = \sqrt{\frac{3}{2}}+\sqrt{\frac{1}{2}}$$

ex find the square root of

$2a+2\sqrt{a^2-b^2} =$

($2a = (a+b)+(a-b)$, $a^2-b^2 = (a+b)(a-b)$)

$$\sqrt{a+b}+\sqrt{a-b}$$

Square root is always positive

eg. $5-2\sqrt{6} = 3+2-2\sqrt{3\times 2}$

($5 = 3+2$, $6 = 3\times 2$)

$= (\sqrt{3})^2+(\sqrt{2})^2-2(\sqrt{3})(\sqrt{2}) = (\sqrt{3}-\sqrt{2})^2$

$$\boxed{\sqrt{5-2\sqrt{6}} = \sqrt{3}-\sqrt{2}}$$

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$10 + \sqrt{24} + \sqrt{60} + \sqrt{90}$ → $2ca$

$2ab$ $2bc$

$(a+b+c)^2 = a^2+b^2+c^2 + 2ab + 2bc + 2ca$

- $a^2+b^2+c^2 = 10$
- $2ab = \sqrt{24} \Rightarrow 2ab = 2\sqrt{6} \Rightarrow ab = \sqrt{6}$ .... (1)
- $2bc = \sqrt{60} \Rightarrow 2bc = 2\sqrt{15} \Rightarrow bc = \sqrt{15}$ .... (2)
- $2ca = \sqrt{40} \Rightarrow 2ca = 2\sqrt{10} \Rightarrow ca = \sqrt{10}$ .... (3)

(1) × (2) × (3)

$(ab)(bc)(ca) = (\sqrt{6})(\sqrt{15})(\sqrt{10}) = \sqrt{6\times15\times10} = 30$

$\Rightarrow (abc)^2 = 30 \Rightarrow abc = \sqrt{30}$ .... (4)

eq'n (4) ÷ eq'n (1) $\quad \frac{abc}{ab} = \frac{\sqrt{30}}{\sqrt{6}} = c = \sqrt{5}$

eq'n (4) ÷ eq'n (2) $\quad \frac{abc}{bc} = \frac{\sqrt{30}}{\sqrt{15}} \Rightarrow a = \sqrt{2}$

eq'n (4) ÷ eq'n (3) $\quad \frac{abc}{ca} = \frac{\sqrt{30}}{\sqrt{10}} \Rightarrow b = \sqrt{3}$

$a = \sqrt{2}$, $b = \sqrt{3}$, $c = \sqrt{5}$

$10 + \sqrt{24} + \sqrt{40} + \sqrt{60} = (\sqrt{2})^2 + (\sqrt{3})^2 + (\sqrt{5})^2 + 2\cdot\sqrt{2}\cdot\sqrt{3} + 2\sqrt{3}\cdot\sqrt{5} + 2\sqrt{5}\cdot\sqrt{2}$

$= (\sqrt{2}+\sqrt{3}+\sqrt{5})^2$

$$\sqrt{10+\sqrt{24}+\sqrt{40}+\sqrt{60}} = \sqrt{(\sqrt{2}+\sqrt{3}+\sqrt{5})^2} = \sqrt{2}+\sqrt{3}+\sqrt{5} \Rightarrow \text{Ans}$$

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$x^{x\sqrt{x}} = (x\sqrt{x})^x$

$\Downarrow$

$x^{3/2} = (x^{3/2})^x$

$x^{3/2} = x^{\frac{3}{2}x}$

$x\sqrt{x} : x \cdot x^{1/2} = x^{1+1/2} = x^{3/2}$

$(a^m)^n = a^{m \times n}$

$a^x = a^y$

$x^{3/2} = \frac{3}{2}x \Rightarrow \frac{x^{3/2}}{x^1} = \frac{3}{2}$

$\Rightarrow x^{\frac{3}{2}-1} = \frac{3}{2}$

$\Rightarrow$

$\Rightarrow x^{1/2} = \frac{3}{2}$

$\Rightarrow \sqrt{x} = \frac{3}{2}$

$x = \frac{9}{4}$

$a^{mn} \quad (a^m)^n = a^{m \times n}$

$a^{mn} \quad (a^m)^n = a^{m \times n}$

$\frac{a^m}{a^n} = a^{m-n}$

$2^{3^4} \quad (2^3)^4$

$\Downarrow$

$2^{81} \quad 2^{12}$

(ii) $2^{x-2} = 5^{2-x}$

$\Rightarrow \frac{2^x}{2^2} = \frac{5^2}{5^x}$

$\Rightarrow 2^x \times 5^x = 5^2 \times 2^2$

$\Rightarrow 10^x = (10)^2$

$x = 2$

(iii) $\frac{2^{x-1} \cdot 2^{2(2x+1)}}{2^{3(x-1)}} = 2^6$

(iv) $2^{x+1} \cdot 5^x = 200$

$\Rightarrow 2^x \cdot 2 \cdot 5^x = 200$ 100

$\Rightarrow (2 \times 5)^x = 100$

$\Rightarrow 10^x = 10^2 \Rightarrow x = 2$

$\left(\sqrt{.0081}\right)^{1/3}$

$\Rightarrow \left(\frac{81}{10000}\right)^{1/3}$

$\Rightarrow \frac{3^5}{10^4}$

$\sqrt{243} = \sqrt{3^5} = (3^5)^{1/2}$

$= 3^{5/2}$

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$$\sqrt[3]{\frac{27^{x-1}}{10000}}$$

$$\frac{(3^{-2})^{2x-1} \times \left(\frac{3^q}{10^q}\right)^{1/3}}{3^{5/2}} = \frac{(3^{-1})^{2x-5} \cdot 3^{x-1}}{10^{q/3}}$$

$$\Rightarrow \left[\frac{(3^3)^{x-1}}{10^q}\right]^{1/3}$$

$$= \frac{3^{3(x-1)(1/3)}}{10^{q/3}}$$

$$\Rightarrow \frac{3^{-2(2x-1)} \times 3^{q/3}}{10^{q/3} \times 3^{5/2}} = \frac{3^{-2x+5} \cdot 3^{x-1}}{10^{q/3}} \times 3^{5/2}$$

$$\Rightarrow 3^{-4x+2+q/3} = 3^{-x+q+5/2}$$

$$-4x + \frac{10}{3} = -x + \frac{13}{2}$$

$$-4x + x = \frac{13}{2} - \frac{10}{3}$$

$$-3x = \frac{39-20}{6}$$

$$-3x = \frac{19}{6} \Rightarrow x = -\frac{19}{18}$$

Qno.

$$\frac{1}{\frac{x^a + x^b + x^c}{x^a}} + \frac{1}{\frac{x^b + x^a + x^c}{x^b}} + \frac{1}{\frac{x^c + x^b + x^a}{x^c}}$$

$$\frac{1}{\frac{x^a + x^b + x^c}{x^a}} + \frac{1}{\frac{x^a + x^b + x^c}{x^b}} + \frac{1}{\frac{x^a + x^b + x^c}{x^c}}$$

$$\Rightarrow \frac{x^a}{x^a + x^b + x^c} + \frac{x^b}{x^a + x^b + x^c} + \frac{x^c}{x^a + x^b + x^c}$$

$$\Rightarrow \frac{x^a + x^b + x^c}{x^a + x^b + x^c} = 1$$

$$\frac{3}{200} + \frac{5}{200}$$

$$\Rightarrow \frac{x^a + x^b + x^c}{x^{-a} + x^{-b} + x^{-c}} = 1$$

$$\frac{2^x}{1+2^x} = \frac{1}{4}$$

$$\Rightarrow \frac{2^x}{1+2^x} = \frac{1}{2^2}$$

$$\Rightarrow 2^x \times 2^2 = 1 + 2^x$$

$$\Rightarrow 2^x \times 4 - 2^x = 1$$

$$\Rightarrow 2^x (4-1) = 1$$

$$\Rightarrow 2^x \times 3 = 1$$

$$2^x = \frac{1}{3}$$

$$\frac{8^x}{1+8^x}$$

$$\Rightarrow \frac{(2^3)^x}{1+(2^3)^x}$$

$$= \frac{(2^x)^3}{1+(2^x)^3} = \frac{\left(\frac{1}{3}\right)^3}{1+\left(\frac{1}{3}\right)^3}$$

$$= \frac{1/27}{1+1/27} = \frac{1/27}{28/27} = \frac{1}{28}$$

$$(a^m)^n = a^{mxn}$$

$$= a^{nxm} = (a^n)^m$$

Sol.

$$\frac{(xyz)^4}{(x^{-2}y^3)^{-3}\left(\frac{1}{z^2}\right)^6} = \frac{x^4 \cdot y^4 \cdot z^4}{x^6 \cdot y^{-9} \cdot z^{-12}}$$

$$= x^{4-6} \cdot y^{4+9} \cdot z^{4+12}$$

$$= x^{-2} \cdot y^{13} \cdot z^{16}$$

$$\frac{1}{z^2} \qquad (z^{-2})^6$$

$$(z^{1/2})^6$$

$$= z^{1/2 \times 6} = z^3$$

$$z^{1/2} \Rightarrow \frac{1}{z^2}$$

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$a+\sqrt{b}$

$(a\times b)^m = a^m \times b^m$

$$\sqrt{\sqrt{50}+\sqrt{98}} = \sqrt{5\sqrt{2}+2\sqrt{12}} = \sqrt{5\sqrt{2}+2\sqrt{6}\cdot\sqrt{2}}$$

$$= \sqrt{\sqrt{2}(5+2\sqrt{6})} = (\sqrt{2})\cdot\sqrt{5+2\sqrt{6}}$$

$$= 2^{1/4}\cdot\sqrt{5+2\sqrt{6}}$$

$3+2$ $\quad$ 3×2

$$= 2^{1/4}(\sqrt{3}+\sqrt{2})$$

$$\sqrt{50}+\sqrt{98} = k^2(5+2\sqrt{6})$$

$$5\sqrt{2}+2\sqrt{12} = k^2(5+2\sqrt{6})$$

$$\Rightarrow 5\sqrt{2}+\sqrt{2}\cdot 2\sqrt{6} = k^2(5+2\sqrt{6})$$

$$\Rightarrow \sqrt{2}(5+2\sqrt{6}) = k^2(5+2\sqrt{6})$$

$$k^2 = \sqrt{2} \Rightarrow k = (2^{1/2})^{1/2} = 2^{1/4}$$

$x^2 = q^2$

$x = q^{1/2}$

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Qno. 9

$$\frac{l}{\sqrt{10}+\sqrt{14}+\sqrt{15}+\sqrt{21}} = \frac{\sqrt{10}-\sqrt{14}-\sqrt{15}+\sqrt{21}}{k}$$

$$\Rightarrow \quad lk = \left[(\sqrt{10}+\sqrt{21})+(\sqrt{14}+\sqrt{15})\right]\left[(\sqrt{10}+\sqrt{21})-(\sqrt{14}+\sqrt{15})\right]$$

$$lk = (\sqrt{10}+\sqrt{21})^2 - (\sqrt{14}+\sqrt{15})^2$$

$$= [10+21+2\sqrt{210}] - [14+15+2\sqrt{210}]$$

$$= 31+2\sqrt{210}-29-2\sqrt{210} = 2$$

$lk = 2$

$l = \frac{2}{k}$

2 | 112

2 | 56

$$11+\sqrt{112} = 11+2\sqrt{28} = \sqrt{7}+\sqrt{4}$$

($7+4$, 7×4)

Qno. 11.

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$x = 2 + 2^{1/3} + 2^{2/3}$ $\qquad x^3 - 6x^2 + 6x = ——$

$\Rightarrow \quad x - 2 = 2^{1/3} + 2^{2/3}$ $\qquad (a+b)^3 = a^3 + b^3 + 3ab(a+b)$

cubing both sides

$(x-2)^3 = (2^{1/3} + 2^{2/3})^3$

$(x-2)^3 = 2 + 2^2 + 3 \cdot 2^{1/3} \cdot 2^{2/3} (2^{1/3} + 2^{2/3})$

$x^3 - 3(x)^2(2) + 3x(2)^2 - (2)^3 = 6 + 3 \cdot 2 (x-2)$

$x^3 - 6x^2 + 12x - 8 = 6 + 6x - 12$

$\Rightarrow x^3 - 6x^2 + 12x - 6x = -12 + 8$

$\Rightarrow x^3 - 6x^2 + 6x = -4$

$(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

$(a-b-c)^2 = a^2 + b^2 + c^2 - 2ab + 2bc - 2ca$

$\sqrt{21 - 4\sqrt{5} + 8\sqrt{3} - 4\sqrt{15}}$

$a^2 + b^2 + c^2 = 21$

$2ab = 4\sqrt{5} \quad \Rightarrow ab = 2\sqrt{5}$

$2bc = 8\sqrt{3} \quad \Rightarrow bc = 4\sqrt{3}$

$2ca = 4\sqrt{15} \quad \Rightarrow ca = 2\sqrt{15}$

$(abc)^2 = 2\sqrt{5} \times 4\sqrt{3} \times 2\sqrt{15}$

$(abc)^2 = 16 \times 15$

$abc = \sqrt{16 \times 15} = 4\sqrt{15}$

$$c = \frac{abc}{ab} = \frac{2\sqrt{15}}{2\sqrt{5}} = 2\sqrt{3}$$

$b =$

$4\overline{)190}\,(35$
12
$\overline{70}$

Sno $\quad \frac{1}{\sqrt{12+\sqrt{140}}}$

$12 + \sqrt{140} = 12 + \sqrt{4\times35}$

$= 12 + 2\sqrt{35}$

$7+5 \quad 7\times5$

$= \frac{1}{\sqrt{12+}}$

$\sqrt{7}+\sqrt{5}$

$8 - \sqrt{60}$

$= 8 - 2\sqrt{15}$

$5+3 \quad 5\times3$

$\sqrt{5}-\sqrt{3}$

$\frac{1}{\sqrt{7}+\sqrt{5}} \quad + \quad \frac{1}{\sqrt{5}-\sqrt{3}}$

$\Rightarrow \frac{\sqrt{5}-\sqrt{3}+\sqrt{7}+\sqrt{5}}{(\sqrt{7}+\sqrt{5})(\sqrt{5}-\sqrt{3})} = 2\sqrt{5}+\sqrt{7}-\sqrt{3}$

$\frac{1}{\sqrt{12+\sqrt{140}}} =$

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$$\frac{2^x}{1+2^x} = \frac{1}{4}$$

$$2^x \cdot 4 = 1 + 2^x$$

$$2^x \cdot 4 - 2^x = 1$$

$$\Rightarrow 2^x (4-1) = 1$$

$$2^x \cdot 3 = 1$$

$$2^x = \frac{1}{3}$$

$$\frac{8^x}{1+8^x}$$

$$8^x = (2^3)^x = 2^{3 \times x} = 2^{x \times 3} = (2^x)^3$$

$$\frac{(2^x)^3}{1+(2^x)^3}$$

$$\frac{(\frac{1}{3})^3}{1+(\frac{1}{3})^3} = \frac{\frac{1}{27}}{1+\frac{1}{28}} = \frac{\frac{1}{27}}{\frac{28}{27}} = \frac{1}{28}$$

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$$\frac{1}{\sqrt{12+\sqrt{140}}} + \frac{1}{\sqrt{8-\sqrt{60}}}$$

$$\Rightarrow \sqrt{12+\sqrt{140}} = \sqrt{12+2\sqrt{35}} = \sqrt{7}+\sqrt{5}$$

($7+5$, 7×5)

$$\sqrt{8-\sqrt{60}} = \sqrt{8-2\sqrt{15}} = \sqrt{5}-\sqrt{3}$$

($5+3$, 5×3)

$$\sqrt{10-\sqrt{84}} = \sqrt{10-2\sqrt{21}}$$

($7+3$, 7×3)

$$= \sqrt{7}-\sqrt{3}$$

$$\frac{1}{\sqrt{7}+\sqrt{5}} + \frac{1}{\sqrt{5}-\sqrt{3}}$$

$$\frac{1}{\sqrt{7}+\sqrt{5}} \times \frac{\sqrt{7}-\sqrt{5}}{\sqrt{7}-\sqrt{5}} + \frac{1}{\sqrt{5}-\sqrt{3}} \times \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}$$

$$\frac{\sqrt{7}-\sqrt{5}}{7-5} + \frac{\sqrt{5}+\sqrt{3}}{5-3}$$

$$\frac{\sqrt{7}-\sqrt{5}}{2} + \frac{\sqrt{5}+\sqrt{3}}{2}$$

$$= \frac{\sqrt{7}-\sqrt{5}+\sqrt{5}+\sqrt{3}}{2}$$

$$= \frac{\sqrt{7}+\sqrt{3}}{2} \times \frac{\sqrt{7}-\sqrt{3}}{\sqrt{7}-\sqrt{3}}$$

$$= \frac{7-3}{2(\sqrt{7}-\sqrt{3})} = \frac{4^2}{2(\sqrt{7}-\sqrt{3})}$$

$$= \frac{2}{\sqrt{7}-\sqrt{3}}$$

$$= \frac{2}{\sqrt{10-\sqrt{84}}}$$

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$$\frac{4+3\sqrt{3}}{\sqrt{7+4\sqrt{3}}}$$

$$\sqrt{7+4\sqrt{3}} = \sqrt{7+2\sqrt{12}} = \sqrt{4}+\sqrt{3} = 2+\sqrt{3}$$

$4+3$ $\quad$ 4×3

$$= \text{[illegible]}$$

$$\frac{4+3\sqrt{3}}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}}$$

$$\frac{(4+3\sqrt{3})(2-\sqrt{3})}{4-3} = \frac{8-4\sqrt{3}+6\sqrt{3}-9}{1} = \frac{-1+2\sqrt{3}}{1} = \sqrt{12}-1$$

a $\quad$ b

$(a, b) = (-1, 12)$

$x = 3-\sqrt{5}$

$$\frac{\sqrt{x}}{\sqrt{2}+\sqrt{3x-2}}$$

$$\sqrt{3x-2}$$

$$= \sqrt{3(3-\sqrt{5})-2}$$

$$= \sqrt{9-3\sqrt{5}-2}$$

$$= \sqrt{7-3\sqrt{5}}$$

$$\sqrt{3-\sqrt{5}} = 3-\frac{2}{2}\sqrt{5}$$

$$= 3-2\sqrt{\frac{5}{4}}$$

$$= 3-2\sqrt{\frac{5}{2}\cdot\frac{1}{2}}$$

$$= \sqrt{\frac{5}{2}}-\sqrt{\frac{1}{2}} = \frac{\sqrt{5}-1}{\sqrt{2}}$$

$$= 7-\frac{2}{2}\sqrt{45}$$

$$= 7-2\sqrt{\frac{45}{4}} = 7-2\sqrt{\frac{9}{2}\times\frac{5}{2}} = \sqrt{\frac{9}{2}}-\sqrt{\frac{5}{2}} = \frac{\sqrt{9}-\sqrt{5}}{\sqrt{2}}$$

$$\frac{\sqrt{x}}{\sqrt{2}+\sqrt{3x-2}} = \frac{\frac{\sqrt{5}-1}{2}}{\sqrt{2}+\frac{\sqrt{9}-\sqrt{5}}{\sqrt{2}}} = \frac{\frac{\sqrt{5}-1}{\sqrt{2}}}{\frac{2+3-\sqrt{5}}{\sqrt{2}}} = \frac{\frac{\sqrt{5}-1}{\sqrt{2}}}{\frac{5-\sqrt{5}}{\sqrt{2}}} = \frac{\frac{\sqrt{5}-1}{\sqrt{2}}\sqrt{2}}{\sqrt{5}(\sqrt{5}-1)} = \frac{1}{\sqrt{5}}$$

Qno. 18.

$$\frac{a+\sqrt{a^2-1}}{a-\sqrt{a^2-1}} \times \frac{a+\sqrt{a^2-1}}{a+\sqrt{a^2-1}}$$

x $\quad$ y $\quad$ x $\quad$ y

$\sqrt{a-\sqrt{a^2-1}}$ (x, y) $\qquad \sqrt{a+\sqrt{a^2-1}}$ (x, y)

$$= \frac{\left(a+\sqrt{a^2-1}\right)^2}{a^2-\left(\sqrt{a^2-1}\right)^2} = \frac{\left(a+\sqrt{a^2-1}\right)^2}{a^2-(a^2-1)}$$

$$= \frac{\left(a+\sqrt{a^2-1}\right)^2}{1} = a^2+(a^2-1)+2a\sqrt{a^2-1}$$

$$= 2a^2-1+2a\sqrt{a^2-1}$$

Que-19. $\dfrac{1}{2-\sqrt{3}} \times \dfrac{2+\sqrt{3}}{2+\sqrt{3}} - \dfrac{1}{\sqrt{3}+\sqrt{2}} \times \dfrac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}} + \dfrac{4}{3-\sqrt{5}} \times \dfrac{3+\sqrt{5}}{3+\sqrt{5}}$

$$= \frac{2+\sqrt{3}}{1} - \frac{(\sqrt{3}-\sqrt{2})}{1} + \frac{4(3+\sqrt{5})}{4}$$

$$= (2+\sqrt{3})-(\sqrt{3}-\sqrt{2})+(3+\sqrt{5}) = 2+\sqrt{3}-\sqrt{3}+\sqrt{2}+3+\sqrt{5}$$

$$= 5+\sqrt{2}+\sqrt{5}$$

20. $\dfrac{4}{(2+\sqrt{3})+\sqrt{7}} \times \dfrac{(2+\sqrt{3})-\sqrt{7}}{(2+\sqrt{3})-\sqrt{7}}$

21. $x = 5+2\sqrt{6}$ $\qquad \sqrt{\dfrac{x}{2}} - \dfrac{1}{\sqrt{2x}} = \dfrac{\sqrt{x}}{\sqrt{2}} - \dfrac{1}{\sqrt{2x}} = \dfrac{\sqrt{3}+\sqrt{2}}{\sqrt{2}} - \dfrac{1}{\sqrt{6}+2}$

$\sqrt{x} = \sqrt{5+2\sqrt{6}} = \sqrt{3}+\sqrt{2}$

$\sqrt{2x} = \sqrt{2}\cdot\sqrt{x} = \sqrt{2}(\sqrt{3}+\sqrt{2})$

$= \sqrt{6}+2$

$$= \frac{\sqrt{3}+\sqrt{2}}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} - \frac{1}{\sqrt{6}+2} \times \frac{\sqrt{6}-2}{\sqrt{6}-2}$$

$$= \left(\frac{\sqrt{6}+2}{2}\right) - \left(\frac{\sqrt{6}-2}{6-4}\right)$$

$$= \frac{(\sqrt{6}+2)-(\sqrt{6}-2)}{2}$$

$$= \frac{\sqrt{6}+2-\sqrt{6}+2}{2} = \frac{4}{2} = 2$$

$3+\sqrt{6}$

$\sqrt{\qquad\qquad 33+19\sqrt{3}}$

(24)

$$\sqrt{\frac{6+2\sqrt{3}}{33-19\sqrt{3}} \times \frac{33+19\sqrt{3}}{33+19\sqrt{3}}}$$

$$= \sqrt{\frac{198+114\sqrt{3}+66\sqrt{3}+114}{(33)^2-(19\sqrt{3})^2}} = \sqrt{\frac{312+180\sqrt{3}}{1089-1083}} = \sqrt{\frac{312+180\sqrt{3}}{6}}$$

$$= \sqrt{\frac{6(52+30\sqrt{3})}{6}}$$

$$= \sqrt{52+2\sqrt{675}}$$

$27+25 \quad 27\times 25$

$$= \sqrt{27}+\sqrt{25} = 5+\sqrt{27} \Rightarrow a+b\sqrt{3}$$

$$= 5+3\sqrt{3}$$

$a+b = 5+3 = 8$

$$\frac{\left(\frac{pq+1}{q}\right)^p \left(\frac{pq-1}{q}\right)^q}{\left(\frac{pq+1}{p}\right)^p \left(\frac{pq-1}{p}\right)^q} = \left(\frac{p}{q}\right)^x$$

$$\Rightarrow \frac{\frac{(pq+1)^p}{q^p} \times \frac{(pq-1)^q}{q^q}}{\frac{(pq+1)^p}{p^p} \times \frac{(pq-1)^q}{p^q}} = \frac{p^p \times p^q}{q^p \times q^q} = \frac{p^{p+q}}{q^{p+q}} = \left(\frac{p}{q}\right)^{p+q}$$

28. $x = \sqrt{\frac{7+4\sqrt{3}}{7-4\sqrt{3}} \times \frac{7+4\sqrt{3}}{7+4\sqrt{3}}} = \sqrt{\frac{(7+4\sqrt{3})^2}{1}} = 7+4\sqrt{3}$

$$\frac{1}{x} = \frac{1}{7+4\sqrt{3}} \times \frac{7-4\sqrt{3}}{7-4\sqrt{3}} = \frac{7-4\sqrt{3}}{1} = 7-4\sqrt{3}$$

$$x + \frac{1}{x} = 7+4\sqrt{3} + 7-4\sqrt{3} = 14$$

29. $3+3\sqrt{3}+5\sqrt{3} = \underline{3}+\underline{8}\sqrt{3}$

31. $a = \frac{1}{3-2\sqrt{2}}$; $b = \frac{1}{3+2\sqrt{2}}$

$$a = \frac{1}{3-2\sqrt{2}} \times \frac{3+2\sqrt{2}}{3+2\sqrt{2}} = \frac{3+2\sqrt{2}}{1}$$

$$b = \frac{1}{3+2\sqrt{2}} \times \frac{3-2\sqrt{2}}{3-2\sqrt{2}} = \frac{3-2\sqrt{2}}{1}$$

$$ab = (3+2\sqrt{2})(3-2\sqrt{2}) = 1$$

$$a+b = 6$$

$$a^2b + ab^2 = ab(a+b)$$

$$= 1(6) = 6$$

$$(a+b)^2 = a^2 + b^2 + 2ab$$

$ab = (3+2\sqrt{2})(3-2\sqrt{2})$

$a+b = 6$

$(a+b)^2 = a^2+b^2+2ab$

$(a+b)^2 - 2ab = a^2+b^2$

32. $x : 7+4\sqrt{3}$ $\quad xy = 1$

$y = \frac{1}{x} = \frac{1}{7+4\sqrt{3}} \times \frac{7-4\sqrt{3}}{7-4\sqrt{3}} = \frac{7-4\sqrt{3}}{1} = 7-4\sqrt{3}$

$(a+b)^2 = a^2+b^2+2ab$

$(a+b)^2 - 2ab = a^2+b^2$

$\Rightarrow \frac{1}{x^2} + \frac{1}{y^2} = \frac{x^2+y^2}{(xy)^2} = \frac{(x+y)^2-2xy}{(xy)^2} = \frac{(14)^2-2(1)}{(1)^2} = 196-2$

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$x^3+y^3+z^3-3xyz = (x+y+z)(x^2+y^2+z^2-xy-yz-zx)$

$\sqrt[3]{a}+\sqrt[3]{b}+\sqrt[3]{c}$

$x \quad y \quad z$

$x+y+z = 0$

$x^3+y^3+z^3-3xyz = 0$

$\Rightarrow x^3+y^3+z^3 = 3xyz$

$a+b+c-3\sqrt[3]{abc} = 0$

$a+b+c = 3\sqrt[3]{abc}$

$\Rightarrow (a+b+c)^3 = 27(abc)$

Ans. $\frac{(x^{a+b})^2 \times (x^{b+c})^2 \times (x^{c+a})^2}{(x^a \cdot x^b \cdot x^c)^4} = \frac{x^{2(a+b)} \cdot x^{2(b+c)} \cdot x^{2(c+a)}}{(x^{a+b+c})^4}$

$= \frac{x^{2a+2b+2c}}{x^{4a+4b+4c}} = 1$

36. $4^x = 5^y = 20^z = K$

$4^x = K \quad ; \quad 5^y = K \quad ; \quad 20^z = K$

$4 = K^{1/x} \quad ; \quad 5 = K^{1/y} \quad ; \quad 20 = K^{1/z}$

$20 = 5 \times 4$

$K^{1/z} = K^{1/y} \times K^{1/x}$

$K^{1/z} = K^{1/x + 1/y}$

$K^{\square} = K^{\square}$

$\frac{1}{z} = \frac{1}{x} + \frac{1}{y}$

$\Rightarrow \frac{1}{z} = \frac{x+y}{xy}$

$\Rightarrow z = \frac{xy}{x+y}$

37. $x^{a^3+b^3+3ab(a+b)} = 2^{125}$

$x^{(a+b)^3} = 2^{125}$

$x^{(5)^3} = 2^{125}$

$x^{125} = 2^{125}$ $\quad x = 2$

38 $\left(x^{\frac{1}{a-b}}\right)^{\frac{1}{c-a}} = x^{\frac{1}{(a-b)(c-a)}} \times x^{\frac{1}{(b-c)(a-b)}} \times x^{\frac{1}{(c-a)\times(b-c)}}$

$= x^{\frac{1}{(a-b)(c-a)} + \frac{1}{(b-c)(a-b)} + \frac{1}{(c-a)(b-c)}} = x^0 = 1$

$\frac{1}{(a-b)(c-a)} + \frac{1}{(b-c)(a-b)} + \frac{1}{(c-a)(b-c)} = \frac{b-c+c-a+a-b}{(a-b)(b-c)(c-a)} = \frac{0}{(a-b)(b-c)(c-a)}$

$= 0$

$(625)^{-\frac{1}{2}\times-\frac{1}{4}\times 2} = (5^4)^{\frac{1}{2}} = 5$

$(625)^{-\frac{1}{2}\times-\frac{1}{4}\times 2} = (5^4)^{\frac{1}{2}} = 5$

$\left[5\left(8^{\frac{1}{3}}+27^{\frac{1}{3}}\right)^3\right]^{\frac{1}{4}} = \left[5(2+3)^3\right]^{\frac{1}{4}} = \left[5\cdot 5^3\right]^{\frac{1}{4}} = (5^4)^{\frac{1}{4}} = 5$

42.

$\left(\frac{729}{10^6}\right)^{-3/4} \times \left(\frac{9}{100}\right)^{-3/4} = \left(\frac{729\times 9}{10^8}\right)^{-3/4}$

$\Rightarrow \left(\frac{3^6}{10^6}\right)^{-3/4} \times = \left(\frac{3^8}{10^8}\right)^{-3/4} = \left[\left(\frac{3}{10}\right)^{8}\right]^{-3/4}$

$= \left(\frac{3}{10}\right)^{-6}$

43. ~~800~~ $^{100} = 8\times 10^8 \times x^{-3/2}$

$\frac{100}{10^8} = x^{-3/2}$

$10^{2-8} = x^{-3/2}$

$\Rightarrow 10^{-6} = x^{-3/2}$ $\Rightarrow \frac{1}{10^6} = \frac{1}{x^{3/2}}$ $\Rightarrow x^{3/2} = 10^6$

$10^6 = x^{3/2}$ $\Rightarrow x = (10^6)$

$x = (10^{6})^{2/3} = 10^4$

44

$\frac{\left(\frac{-5}{6}\right)^{3/4}}{\left(\frac{-5}{6}\right)^{7/6}} = \left(\frac{-5}{6}\right)^{7-x}$

$\Rightarrow \left(\frac{-5}{6}\right)^{\frac{3}{4}-\frac{7}{6}} = \left(\frac{-5}{6}\right)^{7-x}$

$\frac{3}{4}-\frac{7}{6} = 7-x$

$\frac{9-14}{12} = 7-x$

$\frac{-5}{12} = 7-x$

$x = 7 + 5/12 = \frac{89+5}{12} = \frac{89}{12}$

45

$4^x - 4^{x-1} = 24$

$\Rightarrow 4^x - \frac{4^x}{4} = 24$

$\Rightarrow 4^x\left(1 - \frac{1}{4}\right) = 24$

$\Rightarrow 4^x \times \frac{3}{4} = 24^8$

$4^x = 4 \times 8 \Rightarrow 4^x = 32 \Rightarrow (2^{2x}) = 32 \Rightarrow 2^{2x} = 2^5$

$2x = 5 \Rightarrow x = 5/2$

$(2x)^x = \left(2 \times \frac{5}{2}\right)^{5/2} = 5^{5/2}$

$\frac{\left(7^{-1/2} \times 5^2\right)^2}{25^{3/2}} = \frac{7^{-1} \times 5^4}{5^3} = 7^{-1} \times 5 = \frac{5}{7}$

47. $(2d^2e^{-1})^3 \times \left(\frac{d^3}{e}\right)^{-2}$

$8d^6 \times e^{-3} \times \frac{d^{-6}}{e^{-2}} = 8 \times d^{6-6} \times e^{-3+2} = 8 \times 1 \times e^{-1} = \frac{8}{e}$

48 $3 \times 3^m = 3^{3m}$

$3^{m+1} = 3^{3m}$

$3m = m + 1$

$2m = 1 \Rightarrow m = 1/2$

49. $\sqrt{g^x} = \sqrt[3]{g^2}$

$(g^x)^{1/2} = (g^2)^{1/3}$

$g^{x/2} = g^{2/3}$

$\frac{x}{2} = \frac{2}{3} \Rightarrow x = 4/3$

50. $a = x + \frac{1}{x}$ $\qquad x^3 + x^{-3} = x^3 + \frac{1}{x^3}$

$(a)^3 = \left(x + \frac{1}{x}\right)^3$

$a^3 = x^3 + \frac{1}{x^3} + 3(x)\left(\frac{1}{x}\right)\left(x + \frac{1}{x}\right)$

$a^3 = x^3 + \frac{1}{x^3} + 3a$

$a^3 - 3a = x^3 + \frac{1}{x^3}$

Ans 51. $a = x^{1/3} - x^{-1/3} = x^{1/3} - \frac{1}{x^{1/3}}$

$a^3 + 3a = $ ___

$a^3 = \left(x^{1/3} - \frac{1}{x^{1/3}}\right)^3 = (x^{1/3})^3 - \left(\frac{1}{x^{1/3}}\right)^3 - 3(x^{1/3})\left(\frac{1}{x^{1/3}}\right)\left(x^{1/3} - \frac{1}{x^{1/3}}\right)$

$a^3 = x - \frac{1}{x} - 3a$

$a^3 + 3a = x - 1/x$

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Qno. 53. $a^x = b^y = c^z$; $abc = 1$

$xy + yz + zx$

let $a^x = b^y = c^z = k$

$\Rightarrow a = k^{1/x}$; $b = k^{1/y}$; $c = k^{1/z}$

As $abc = 1$

$k^{1/x} \cdot k^{1/y} \cdot k^{1/z} = 1$

$k^{\frac{xy+yz+zx}{xyz}} = k^0$

$\Rightarrow \frac{xy+yz+zx}{xyz} = 0 \Rightarrow$ $xy + yz + zx = 0$

$\frac{a^x}{a} = bc$; $b^y = abc$ $c^z = abc$

$a^x = abc$

$a^x = b^y = c^z = abc = k$

let $a^x = k$; $b^y = k$; $c^z = k$ & $abc = k$

$a = k^{1/x}$; $b = k^{1/y}$; $c = k^{1/z}$ $abc = k$

$k^{1/x + 1/y + 1/z} = k^1 \Rightarrow \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1$

$\frac{xy+yz+zx}{xyz} = 1$

$\Rightarrow xy + yz + zx = xyz$

$xy + yz + zx - xyz = 0$ ✓

$xy + yz + zx - xyz = 0$ ✓✓

55. $a^{m \times n} = a^{m^n}$

$mn = m^n$

$n = \frac{m^n}{m}$

$\Rightarrow m^{n-1} = n$

$m = (n)^{1/n-1}$

56. $\left(\frac{q}{p}\right)^{1-2x} = \sqrt{\frac{p}{q}}$

$\left(\frac{q}{p}\right)^{1-2x} = \left[\left(\frac{q}{p}\right)^{-1}\right]^{1/2}$

$\left(\frac{q}{p}\right)^{1-2x} = \left(\frac{q}{p}\right)^{-1/2}$

$1 - 2x = -1/2$

$\Rightarrow 1 + 1/2 = 2x$

$3/2 = 2x$

$\Rightarrow x = 3/4$

57. $\frac{a^{q-p}}{a^1}$